Whisker Length Measurement: a Validation Study

Objectives

• To provide step-by-step instructions in whisker length measurement from two images
• To collect data for whisker length measured by different participants to see whether the method provides consistent results regardless of who is using it
Tin Whisker: an Introduction

- **Tin Whisker** - conductive crystalline structure of tin growing outward from tin rich surfaces
- Whiskers are formed through addition of atoms at the base, not the tip
  - Lengths vary from few micrometers to millimeters
  - Thicknesses range typically 0.5-10µm
  - Whisker densities may range from just a few whiskers to thousands per component
  - This process may take hours, days, or years
- Long range diffusion responsible for tin transfer to site of whisker growth
- Types of Failures induced by Whiskers:
  - Electrical short circuit
    - *Permanent* if Current < Melting Current
    - *Intermittent* if Current > Melting Current
  - Metal Vapor Arc
    - Applications with high levels of current and voltage may cause whisker vaporizing into conductive plasma of metal ions
    - Plasma forms an arc capable of sustaining hundreds of amps
Whisker Length Definition

JESD22-A121 (May 2005)
The distance between the finish surface and the tip of the whisker that would exist if the whisker were straight and perpendicular to the surface

JESD201 (March 2006)

JESD22-A121A (July 2008)

IEC 60068-2-82 (May 2007)
The straight line distance from the point of emergence of the whisker to the most distant point on the whisker

“Segmented Length”

A + B + C = whisker length

“Shorting Length”

Radius of Sphere = whisker length
3D Nature of Whiskers

- Whiskers are 3-dimensional structures.
- A single view may not provide enough information to measure whisker’s true length, unless you have positioned the whisker perpendicular to the viewing direction.
- Current industrial standards suggest to tilt the whisker until it has been positioned perpendicular to the field of view, and its true length is visible.

HOWEVER…

Same whisker viewed from two different angles.
Practicality Issue

- Too many whiskers to be tilting each one
- Some whiskers exhibit complicated geometries
- Geometry of sample may not allow much degree of freedom
- Nevertheless, any modeling of whisker length requires a statistically significant number of whiskers to be measured. Thus, a more practical approach is needed.
Recommended Length Measurement

A more accurate measurement can be made by using two images offset by a known tilt.

Axis along $L_{ac}$ is the tilt axis.

$L_{cd} = \text{projection of whisker length on axis perpendicular to tilt axis in Plane 1}$

$L_{ce} = \text{projection of whisker length on axis perpendicular to tilt axis in Plane 2}$

$\theta = \text{tilt angle between Plane 1 and Plane 2}$

$\beta = \text{angle between } L_{cd} \text{ and } L_{ad} \text{ in Plane 1}$

$$L_{ab} = \sqrt{\frac{L_{cd}^2 + L_{ce}^2 - 2L_{cd}L_{ce}\cos\theta}{\sin^2\theta}} + (L_{cd}\tan\beta)^2$$

See Backup slides for step-by-step derivation.
Whisker Measurement Example

- X-axis is the tilt axis
- 4 things needed:
  - Tilt angle $\theta = 10^\circ$
  - $L_{cd} = 63\mu m$
  - $L_{ce} = 83\mu m$
  - Angle $\beta$ (taken from 0° tilt picture) = 38°
Whisker Measurement Example

From previous slide:

\[ L_{ab} = \sqrt{\frac{L_{cd}^2 + L_{ce}^2 - 2L_{cd}L_{ce} \cos \theta}{\sin^2 \theta} + (L_{cd} \tan \beta)^2} \]

\[ L_{ab} = \sqrt{\frac{(66)^2 + (83)^2 - 2(66)(83) \cos 10}{\sin^2 10} + (66 \tan 38)^2} \]

\[ L_{ab} = 133 \mu m \]

• If we were to measure only the length of a whisker from a single picture \((L_{ad})\), the length of whisker would be approximated as 83\(\mu m\). Resulting in an error of 40%.

• The error in measurements would differ depending on the angle of whisker growth, with error being larger the larger is the angle between whisker and the surface it grows from.
Goal of This Study

- To demonstrate the ease and repeatability in whisker length measurement using the method described above
- Multiple participants will be introduced to whisker measurement method and asked to measure whisker length from given two pictures of the same whisker
- Total of 15 whiskers to be measured
- First whisker – the length to be measured will be indicated by a line
- Several whiskers will be simple single-segment straight whiskers
- Several whiskers will be of complicated geometry – the participants will be asked measure the whisker by either the segmented or shorting length methods
Tutorial
Image J Software

• In order to conduct whisker measurements from picture, you will first need download a free software for image analysis
  
  http://rsbweb.nih.gov/ij/download.html

• Once installed and opened, you will see the following toolbar

  ![Image J Software toolbar]

  Line tool will be used from here on
Practice 1 (1/7)

• Let us re-measure the whisker that was shown in demonstration a few slides ago
• With this tutorial you should have received a collection of images to be measured
  – Open files “Practice1-tilt0.jpg” and “Practice1-tilt10.jpg”
  – Use “–” in order to make images smaller, “+” to make them larger
  – Hold “Space” key and click+drag the image in order to move around
Practice 1 (2/7)

• Select the Line tool
• Setting the Scale:
  – The program will need to know the scale of your picture (pixels to micrometer conversion)
  – On either of the images, move towards the databar, then click and drag in order to outline a yellow line
  – The line needs to be perfectly horizontal, verify that by monitoring “angle” on the toolbar. Once angle=0.00, the line is horizontal
Practice 1 (3/7)

- Now that the line is set for the scale, go to Analyze -> Set Scale
- Input the “Known Distance” to be the length measured by the bar (50µm in this case)
- Check “Global”
  - This will ensure that all the images that are currently open have the same scale
  - This will be useful for all image pairs here, since each pair has been taken at the same magnification – you will not need to re-measure the databar for each image
  - Note: you will need to set scale for each new pair of images, since different pairs are taken at different magnification levels
Practice 1 (4/7)

- Along with this tutorial and practice images, you should have received an Excel spreadsheet “formula.xls”

* File Name: “Practice 1”

<table>
<thead>
<tr>
<th>File Name</th>
<th>( \alpha ) with tilt 0° in * (input)</th>
<th>( L ) (tilt 0°) in um (input)</th>
<th>( \alpha ) with tilt 10° in * (input)</th>
<th>( L ) (tilt 10°) in um (input)</th>
<th>( \beta ) in * (L12) (input)</th>
<th>( \beta ) in * (L14) (input)</th>
<th>( \theta ) in °, Change between 2 pictures (input)</th>
<th>( L_{12 \text{ in um}} ) (L12)</th>
<th>( L_{14 \text{ in um}} ) (L14)</th>
<th>( L_{01 \text{ in um}} ) (L01)</th>
<th>( L_{ab \text{ in um}} ) (Lab)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#VALUE!</td>
<td>#VALUE!</td>
<td>#VALUE!</td>
<td>#VALUE!</td>
<td>#VALUE!</td>
<td>#VALUE!</td>
<td>#VALUE! (input)</td>
<td>#VALUE!</td>
<td>#VALUE!</td>
<td>#VALUE!</td>
<td>#VALUE!</td>
</tr>
</tbody>
</table>
Practice 1 (5/7)

- Read before measuring
  - All measurements done in straight lines
  - The measurements on both images have to be done between the same points
  - Click at point of origin and drag until reaching the length you want to measure
  - Always start the line at the point that is lower in the picture, and go to point that is higher
Practice 1 (6/7)

• With line in place
  – Analyze -> Measure
  – “Results” window will appear with Length and Angle of the line
   (Angle is measured from horizontal)
• The angle and length needs to be input as well as the tilt angle (10° in this case)

<table>
<thead>
<tr>
<th>File Name</th>
<th>α with tilt 0° in °</th>
<th>L (tilt 0°) in um</th>
<th>α with tilt 10° in °</th>
<th>L (tilt 10°) in um</th>
<th>β in ° (L12)</th>
<th>β in ° (L14)</th>
<th>Lce in um</th>
<th>Lcd in um</th>
<th>θ in ° Change between 2 pictures</th>
<th>Lbc in um</th>
<th>Lab in um</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practice 1</td>
<td>122</td>
<td>97.7</td>
<td>128</td>
<td>83.5</td>
<td>32.00</td>
<td>38.00</td>
<td>82.85</td>
<td>65.80</td>
<td>10</td>
<td>123.05</td>
<td>133.49</td>
</tr>
</tbody>
</table>
Practice 1 (7/7)

• With the marking line showing:
  – Analyze -> Label

• This will place a label on the line you have just measured

• You will need to SAVE this image with the label on it to show which line you have measured
Instructions

• For each image, Practice 1 – Practice 15, conduct either segmented or shorting length measurement (which method to use is indicated on each image)

• Record angle and length for each segment with corresponding file name in formula.xls
  – If this is a multi-segmented whisker, each segment needs to have its own line

• Label the measurement lines and save the images with these labels

• Send the Excel file and labeled JPEG images to
  lyudmyla@calce.umd.edu
Backup: Derivation of Whisker Length formula
A single view of the whisker under a microscope.
Only observing a projection of whisker – $L_{cd}$
Tilting microscope stage $\theta$ degrees
Now observing a different projection of whisker - $L_{ce}$
Known:
\( L_{cd} \) – whisker projection at first view
\( L_{ce} \) – whisker projection after a tilt of \( \theta \)
\( \theta \) – angle of stage tilt
Known:
$L_{cd}$, $L_{ce}$, $\theta$

$L_{cx} = \frac{L_{cd}}{\cos \theta}$

$L_{xe} = L_{ce} - L_{cx} = L_{ce} - \frac{L_{cd}}{\cos \theta}$

$L_{be} = \frac{L_{xe}}{\tan \theta} = \frac{L_{ce} - \frac{L_{cd}}{\cos \theta}}{\tan \theta}$

$L_{bc}^2 = L_{ce}^2 + L_{be}^2 = L_{ce}^2 + \left(\frac{L_{ce} - \frac{L_{cd}}{\cos \theta}}{\tan \theta}\right)^2$
Known:
\( L_{cd}, L_{ce}, \theta \)

\[
L_{bc}^2 = L_{ce}^2 + L_{be}^2 = L_{ce}^2 + \left( \frac{L_{ce} - L_{cd}}{\cos \theta} \right)^2
\]

\[
L_{bc} = \sqrt{L_{ce}^2 + \left[ \frac{L_{ce} - L_{cd}}{\cos \theta} \right]^2}
\]

Rearrange Expression:

\[
L_{bc} = \frac{1}{\sin \theta} \sqrt{L_{cd}^2 + L_{ce}^2 - 2L_{cd}L_{ce} \cos \theta}
\]
However, this is true if the whisker lies perpendicular to the tilt axis: a 2D case

\[ L_{bc} = \frac{1}{\sin \theta} \sqrt{L_{cd}^2 + L_{ce}^2 - 2L_{cd} L_{ce} \cos \theta} \]
Expanding to 3D

Assuming whisker in a single plane perpendicular to the tilt axis

Whisker oriented in 3D space
Construct a rectangular-base pyramid from whisker projections

Known:
\[ L_{cd}, L_{ce}, \theta, \beta, \]

\[ L_{bc} = \frac{1}{\sin \theta} \sqrt{L_{cd}^2 + L_{ce}^2 - 2L_{cd}L_{ce} \cos \theta} \]

\[ L_{bd}^2 = L_{bc}^2 - L_{cd}^2 = \frac{L_{cd}^2 + L_{ce}^2 - 2L_{cd}L_{ce} \cos \theta}{\sin^2 \theta} - L_{cd}^2 \]

\[ L_{ad} = \frac{L_{cd}}{\cos \beta} \]

\[ L_{ab}^2 = L_{bd}^2 + L_{ad}^2 \]

\[ L_{ab}^2 = \left( \frac{L_{cd}^2 + L_{ce}^2 - 2L_{cd}L_{ce} \cos \theta}{\sin^2 \theta} - L_{cd}^2 \right) + \left( \frac{L_{cd}}{\cos \beta} \right)^2 \]

\[ L_{ab}^2 = \left( \frac{L_{cd}^2 + L_{ce}^2 - 2L_{cd}L_{ce} \cos \theta}{\sin^2 \theta} \right) + L_{cd}^2 \left( \frac{1}{\cos^2 \beta} - 1 \right) \]

\[ L_{ab}^2 = \left( \frac{L_{cd}^2 + L_{ce}^2 - 2L_{cd}L_{ce} \cos \theta}{\sin^2 \theta} \right) + L_{cd}^2 \tan^2 \beta \]

Trig identity
Whisker Measurement Equation

\[ L_{ab} = \sqrt{\frac{L_{cd}^2 + L_{ce}^2 - 2L_{cd} L_{ce} \cos \theta}{\sin^2 \theta}} + (L_{cd} \tan \beta)^2 \]

Axis along \( L_{ac} \) is the tilt axis

- \( L_{cd} = \) projection of whisker length on axis perpendicular to tilt axis in Plane 1
- \( L_{ce} = \) projection of whisker length on axis perpendicular to tilt axis in Plane 2
- \( \theta = \) tilt angle between Plane 1 and Plane 2
- \( \beta = \) angle between \( L_{cd} \) and \( L_{ad} \) in Plane 1